Recent progress in (baryon) chiral perturbation theory

Matthias R. Schindler

The George Washington University

8th European Research Conference on
Electromagnetic Interactions with Nucleons and Nuclei

September 27 - October 2, 2009

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON DC

Effective Field Theories

Low-energy approximation to a more fundamental theory

- Description in terms of relevant degrees of freedom (e.g. pions, nucleons,...)
- Most general Lagrangian consistent with all symmetries of underlying theory $\rightarrow$ Ward identities satisfied
- Expansion in q/Λ, where
  - q: momenta and masses
  - Λ: energy scale
  - $q \ll \Lambda$
- Not renormalizable in traditional sense, but infinities can be absorbed in coefficients of the Lagrangian up to arbitrary order

Baryon chiral perturbation theory

- Effective field theory of QCD at low energies
- Typical scale: $\Lambda \approx 1$ GeV (m_ρ, m_N, $4\pi F_\pi$)
- Interaction of pions with nucleons and external fields
- Approximate chiral symmetry: Spontaneously broken
  $\Rightarrow$ Goldstone bosons (pions)
- q: pion masses and external momenta $\ll \Lambda$
- Organization of the Lagrangian in the number of (covariant) derivatives acting on Goldstone boson fields and in the number of quark mass terms

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \cdots \\ &= \mathcal{L}_2 + \mathcal{L}_4 + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots\end{aligned}$$

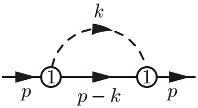
Power counting

Scheme to decide on relative importance of diagrams

- Each diagram is assigned chiral order D
- Renormalized diagram is of order q^D
  - Loop integration in n dimensions $\sim \mathcal{O}(q^n)$
  - Vertex from $\mathcal{L}_{\pi N}^{(i)} \sim \mathcal{O}(q^i)$
  - Nucleon propagator $\sim \mathcal{O}(q^{-1})$
  - Pion propagator $\sim \mathcal{O}(q^{-2})$
- Diagrams with higher D are less important
- Relation between chiral and loop expansion

$D \leq 4 \longrightarrow$ one-loop calculation

Example

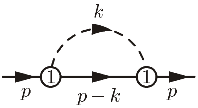


- Power counting: $D = n + 2 \cdot 1 - 1 - 2 = n - 1 \xrightarrow{n \to 4} 3$
- Apply $\overline{\text{MS}}$ renormalization: $\Sigma_r \sim \mathcal{O}(q^2) \Leftrightarrow D = 2$

No consistent power counting for baryonic sector?

Nucl. Phys. B **307**, 779 (1988)

Example



- Power counting: $D = n + 2 \cdot 1 - 1 - 2 = n - 1 \overset{n \to 4}{\longrightarrow} 3$
- Apply $\overline{\text{MS}}$ renormalization: $\Sigma_r \sim \mathcal{O}(q^2) \Leftrightarrow D = 2$

No consistent power counting for baryonic sector?

Gasser, Sainio, Švarc

> *The fact that higher order loops start to contribute at order q^2 of course only means that the relevant low-energy coupling constants at order $q^2, q^3, \ldots$ are renormalized by those loops. The same phenomenon would occur in the mesonic sector if one did not make use of dimensional regularization.*

Nucl. Phys. B **307**, 779 (1988)

Heavy-baryon ChPT

- Separate nucleon momentum into large part close to mass-shell and small residual piece

$$p_\mu = m v_\mu + k_\mu$$

- Perform additional $1/m$ expansion (similar to Foldy-Wouthuysen)
- 'Standard' $\widetilde{\text{MS}}$ as in mesonic sector leads to consistent power counting
- Variety of physical quantities calculated in HBChPT
- Large number of terms at higher-orders
- Wrong analytical behavior in limited number of specific kinematic regimes

---

Jenkins, Manohar, Phys. Lett. B **255**, 558 (1991)

Covariant formulations of BChPT

- Expansion in $1/m$ not necessary
- Suitable choice of renormalization scheme leads to consistent power counting
- Infrared regularization
  - Subtract closed form expression of integrals $\hat{=}$ infinite number of terms
  - Introduces unphysical cuts at high energies
  - In original formulation only applicable to one loop diagrams with pion and nucleon lines
  - Can be reformulated and extended to multi-loop diagrams and additional degrees of freedom
- Extended on-mass-shell scheme (EOMS)
  - Only subtract those terms that violate the power counting (finite number)
  - Can be applied to multi-loop diagrams and diagrams with arbitrary degrees of freedom

---

Becher, Leutwyler, Eur. Phys. J. C **9**, 643 (1999)
MRS, Gegelia, Scherer, Phys. Lett. B **586**, 258 (2004)
Fuchs, Gegelia, Japaridze, Scherer, Phys. Rev. D **68**, 056005 (2003)

Chiral extrapolations

Chiral perturbation theory:

- Expansion in quark masses
- Natural tool to perform extrapolations of lattice results to physical quark masses
- Allows for more systematic error estimate

Convergence

- Expansion parameter q/Λ
- For $q \sim M_\pi, \Lambda \sim M_\rho$: $q/\Lambda \sim 20\%$
- Even and odd orders: convergence slower than in mesonic sector
- Axial coupling g_A: correction at order $M^3 \approx 30\%$

Nucleon mass at $\mathcal{O}(q^6)$

Nucleon mass

- Simplest quantity
- Of interest for chiral extrapolations
- At $\mathcal{O}(q^6)$ contributions from
  - Tree-level diagrams with vertices up to order $\mathcal{O}(q^6)$
  - One-loop diagrams with vertices up to order $\mathcal{O}(q^4)$
  - Two-loop integrals with vertices up to order $\mathcal{O}(q^2)$
- Need renormalization of two-loop diagrams

MRS, Gegelia, Scherer, Nucl. Phys. B **682**, 367 (2004)

Chiral expansion to $\mathcal{O}(q^6)$

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4$$
$$+ k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6$$

Examples:

$$k_5 = \frac{3g_A^2}{1024\pi^3 F^4}\left(16 g_A^2 - 3\right)$$
$$k_6 = \frac{3g_A^2}{256\pi^3 F^4}\left[g_A^2 + \frac{\pi^2 F^2}{m^2} - 8\pi^2(3l_3 - 2l_4) - \frac{32\pi^2 F^2}{g_A}(2d_{16} - d_{18})\right]$$

MRS, Djukanovic, Gegelia, Scherer, Nucl. Phys. A **803**, 68 (2008)

Numerical estimates

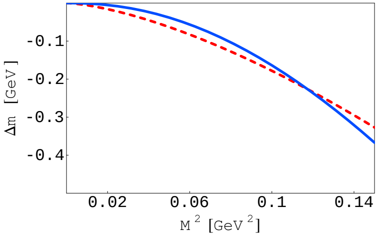
k_5

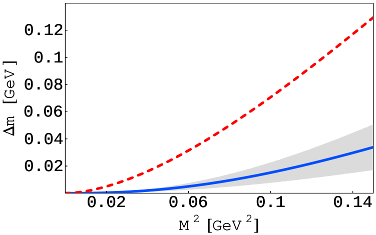
- Free of unknown low-energy constants
- $k_5 M^5 \ln(M/m_N) = -4.8\,\text{MeV}$
- $\approx 30\%$ of leading nonanalytic contribution at one-loop order, $k_2 M^3$

k_6

- l_3, l_4 known
- d_{18} from Goldberger-Treiman discrepancy
- d_{16} not reliably determined: $\pi N \to \pi\pi N$ or lattice fit
- $k_6 M^5 = 3.7\,\text{MeV}$ or $k_6 M^5 = -7.6\,\text{MeV}$

Chiral extrapolations





dashed: $k_2 M^3$
solid: $k_5 M^5 \ln(M/m_N)$

dashed: $k_3 M^4 \ln(M/m_N)$
solid: $k_7 M^6 \ln^2(M/m_N)$

- $k_5 M^5 \ln(M/m_N)$ larger than $k_2 M^3$ for $\sim$ 370MeV?
- Similar limit of applicability as obtained from other estimates (nucleon mass, axial coupling g_A)

Djukanovic, Gegelia, Scherer, Eur. Phys. J. A **29**, 337 (2006)
Bernard, Meißner, Phys. Lett. B **639**, 278 (2006)

Nucleon electromagnetic form factors

- Dirac and Pauli form factors defined via

$$\langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_N}F_2(Q^2)\right]u(p),$$

$$q_\mu = {p'}_\mu - p_\mu, \quad Q^2 = -q^2$$

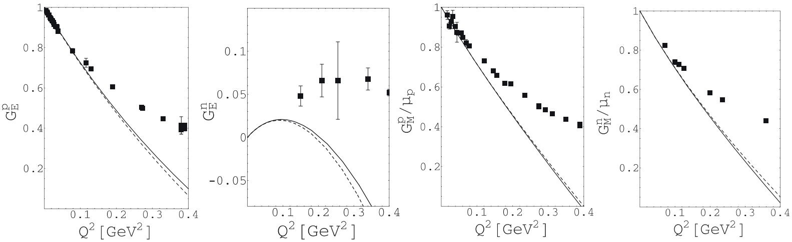
- Sachs form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_n^2}F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$$

Electromagnetic form factors in BChPT

- In manifestly Lorentz-invariant BChPT at order $\mathcal{O}(q^4)$:



- Missing curvature $\longrightarrow$ Higher-order terms

Kubis, Meißner, Nucl. Phys. A **679**, 698 (2001)
MRS, Gegelia, Scherer, Eur. Phys. J. A **26**, 1 (2005)

Additional dynamical degrees of freedom

- Vector meson dominance: important contribution to form factors also in BChPT
- Heavy degrees of freedom in standard ChPT
  - Do not appear explicitly
  - Expand propagator in small momenta

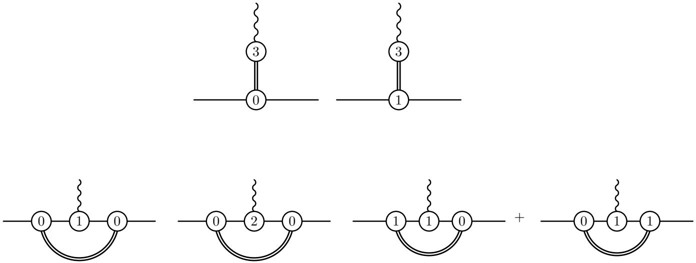
$$\frac{1}{q^2 - M_V^2} = -\frac{1}{M_V^2}\left[1 + \frac{q^2}{M_V^2} + \left(\frac{q^2}{M_V^2}\right)^2 + \left(\frac{q^2}{M_V^2}\right)^3 + \mathcal{O}(q^8)\right]$$

  - Contributions absorbed in coupling constants at each order
- Treat as explicit degrees of freedom
- Resummation of some higher-order terms

Kubis, Meißner, Nucl. Phys. A **679**, 698 (2001)

Inclusion of vector mesons

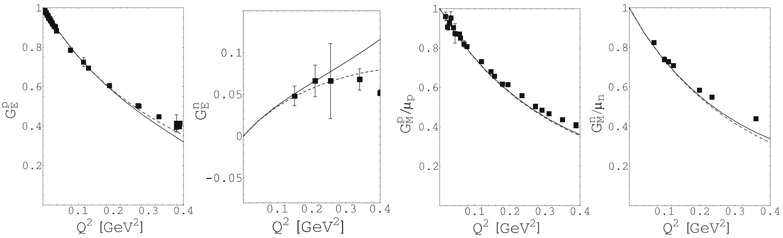
Diagrams to order $\mathcal{O}(q^4)$



- Tree diagrams contribute to $F_1(q^2)$ and $F_2(q^2)$
- Loop diagrams: Only nucleon and vector meson propagators
  $\Rightarrow$ vanish in IR renormalization
- Power counting + renormalization
  $\Rightarrow$ Vector meson loops strongly suppressed

Electromagnetic form factors with vector mesons

- Improved description by inclusion of vector mesons (ρ, ω, ϕ)



- Vector meson coupling constants taken from dispersion relations

---

MRS, Gegelia, Scherer, Eur. Phys. J. A **26**, 1 (2005) [cf. Kubis, Meißner, Nucl. Phys. A **679**, 698 (2001)]

Complex-mass renormalization

- Extend effective field theory to include other resonances
- Unstable states $\Rightarrow$ power counting?
- Apply complex-mass renormalization
- Application to vector mesons, Roper, ...

Denner, Dittmaier, Roth, Wackeroth, Nucl. Phys. B **560**, 33 (1999)
Djukanovic, Gegelia, Keller, Scherer, Phys. Lett. B **680**, 235 (2009)
Djukanovic, Gegelia, Scherer, arXiv:0903.0736 [hep-ph]

Chiral extrapolations of form factors

- Chiral expansion of form factors given by

$$F_1^p(Q^2) = 1 + a_1 Q^2 + a_2 Q^4 + a_3 M^2 Q^2 + \ldots\ldots$$

- Simultaneous expansion in Q^2 and M^2
- For low Q^2 terms like $Q^6 M^2 \sim \mathcal{O}(q^8)$ are suppressed
- For high Q^2 would need to resum an infinite number of formally higher-order terms
- Domain of applicability: $q \lesssim 350\text{MeV}$?

Compton scattering

Compton scattering on proton

- Access to proton (spin) polarizabilities
- see V. Pascalutsa

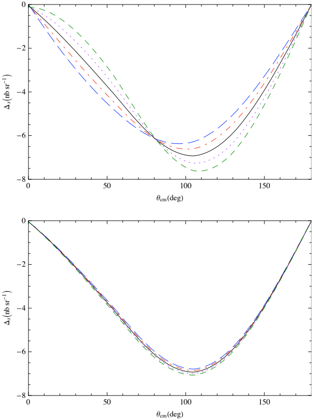
Neutron (spin) polarizabilities

- Use ^{3}He to extract neutron polarizabilities
- Advantage: differential cross section large compared to deuteron case
- HBChPT can be used in EFT calculations in few-body systems (χET)
- One-body operators from HBChPT
- Wave functions and two-body operators derived in EFT formalism
- First theoretical treatment of Compton scattering on ^{3}He performed in χET
- Current results: $\mathcal{O}(q^3)$, no Δ
- Inclusion of Δ, extension to $\mathcal{O}(q^4)$ are ongoing

---

Shukla, Nogga, Phillips, Nucl. Phys. A **819**, 98 (2009)

Double-polarization observable Δ_x in ^{3}He (120 MeV)



from top left to bottom right: vary $\gamma_{i,n}$, $i = 1, 2, 3, 4$

$\vec{\gamma}^{3}\vec{\mathrm{He}} \rightarrow \gamma^{3}\mathrm{He}$

- Double-polarization observables Δ_z, Δ_x sensitive to spin polarizabilities $\gamma_1, \gamma_2, \gamma_4$
- Sensitivity to different combination
- Combine with results from Compton scattering on deuteron
- Curves look very similar to $\vec{\gamma}\vec{n} \rightarrow \gamma n$: polarized ^{3}He as neutron target
- To be measured at HIγS

Choudhury, Phillips, Phys. Rev. C **71**, 044002 (2005)
Hildebrandt, Griesshammer, Hemmert, Phillips, Nucl. Phys. A **748**, 573 (2005)
Hildebrandt, Griesshammer, Hemmert, arXiv:nucl-th/0512063

SU(3) ChPT

- Can consider strange quark as small parameter
- SU(3) ChPT: baryon octet and octet of pseudo-Goldstone bosons
- Convergence?
  - Baryon masses to $\mathcal{O}(q^4)$ in EOMS scheme: large cancellations between different orders
  - Baryon magnetic moments in EOMS scheme: good convergence
- Further studies needed

Lehnhart, Gegelia, Scherer, J. Phys. G **31**, 89 (2005)
Geng, Camalich, Alvarez-Ruso, Vacas, Phys. Rev. Lett. **101**, 222002 (2008)

Summary and Outlook

Baryon chiral perturbation theory

- Effective field theory for hadronic processes at energies $E \ll 1\,\mathrm{GeV}$
- Applied to a variety of processes (static properties, πN, electromagnetic,...)
- Model-independent
- Systematic error estimation
- Natural tool for chiral extrapolations of lattice data
- As all EFTs: limited domain of applicability ($M_\pi \lesssim 350\mathrm{MeV}$)

Current developments

- Calculations up to order $\mathcal{O}(q^6)$ ($\hat{=}$ two-loop level)
- Extension to higher-energies by inclusion of additional degrees of freedom
- Explicit Δ degrees of freedom
- Results now used as input in few-body calculations
- Few-body calculations for neutron properties (e.g. polarizabilities from ^{3}He)
- SU(3): convergence problematic?
- Framework for study of isospin symmetry breaking

Compton scattering

- Allows to study sub-structure of nucleon
- At low photon energies can write effective Hamiltonian

$$H_{eff} = \frac{(\mathbf{p} - Q\mathbf{A})^2}{2m} + Q\Phi - \frac{1}{2}4\pi(\alpha\mathbf{E}^2 + \beta\mathbf{H}^2 + \gamma_{E1E1}\boldsymbol{\sigma}\cdot\mathbf{E}\times\dot{\mathbf{E}}$$
$$+ \gamma_{M1M1}\boldsymbol{\sigma}\cdot\mathbf{H}\times\dot{\mathbf{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j)$$

where $F_{ij} = \frac{1}{2}\left(\nabla_i F_j + \nabla_j F_i\right)$

- α/β: electric/magnetic polarizabilities
  γ_{XY}: spin-dependent polarizabilities
- Describe response of object to external e.m. field
- Can be probed in Compton scattering
- Note: BChPT gives information on cross-section, not just polarizabilities

Proton polarizabilities in HBChPT

- Up to (and including) NLO ($\mathcal{O}(q^3)$) only low-energy constants are g_A, f, m (units of 10^{-4} fm^3)

$$\alpha_p = 12.2, \beta_p = 1.2$$

- At NNLO ($\mathcal{O}(q^4)$) new LECs $\Rightarrow$ fit to data

$$\alpha_p = (12.4 \pm 1.1)^{+0.5}_{-0.5},\ \beta_p = (3.4 \pm 1.1)^{+0.1}_{-0.1}$$

- Baldin sum rule constrained fit

$$\alpha_p = (11.2 \pm 0.2)^{+0.5}_{-0.5},\ \beta_p = (2.8 \pm 0.5 \mp 0.2)^{+0.1}_{-0.1}$$

- PDG values (using dispersion relations)

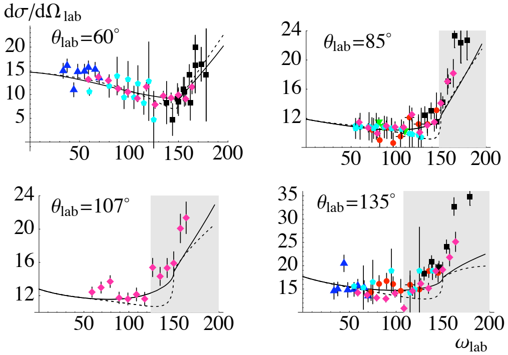
$$\alpha_p = 12.0 \pm 0.6, \beta_p = 1.9 \pm 0.5$$

- Success?

V. Bernard, N. Kaiser and U. G. Meissner, Phys. Rev. Lett. **67**, 1515 (1991)
S. R. Beane, M. Malheiro, J. A. McGovern, D. R. Phillips and U. van Kolck, Phys. Lett. B **567**, 200 (2003)

Δ contributions

- Comparison with Compton scattering differential cross section



- Deviation from data at higher photon energies $\rightarrow$ Δ contributions
- Inclusion of Δ at order $\mathcal{O}(q^3)$: $\alpha_p \approx 17$, $\beta_p \approx 13$
- Fixes: "Demote" some Δ contributions, "promote" LECs

S. R. Beane *et al*, Phys. Lett. B **567**, 200 (2003)
R. P. Hildebrandt *et al*, Eur. Phys. J. A **20**, 329 (2004)

Polarizabilities in BChPT

- First calculation of polarizabilities performed in BChPT
- At order $\mathcal{O}(q^3)$

$$\alpha_p = 6.8,\ \beta_p = -1.8$$

- Blessing in disguise?
- Large Δ contributions can be accommodated without problems
- At order $\mathcal{O}(p^4/\Delta)$

$$\alpha_p = 10.8 \pm 0.7 \pm \ldots,\ \beta_p = 4.0 \pm 0.7 \pm \ldots$$

- But: very good description of differential cross section
- Precision measurements at very low energies?
- Calculation at order $\mathcal{O}(q^4)$ (also VCS) is finished

---

V. Bernard, N. Kaiser and U. G. Meissner, Phys. Rev. Lett. **67**, 1515 (1991)
V. Lensky and V. Pascalutsa, arXiv:0907.0451 [hep-ph]
D. Djukanovic, PhD thesis (2008)

Neutron polarizabilities

- Prediction of (H)BChPT to order $\mathcal{O}(q^3)$

$$\alpha_p = \alpha_n,\ \beta_p = \beta_n,\ \gamma_{i,p} = \gamma_{i,n}$$

- No free neutron target:
  - Neutron photo-production: $\alpha_n + \beta_n = 15.2 \pm 0.5$
  - Neutron scattering on lead: $\begin{cases} \alpha_n = 12.6 \pm 1.5 \pm 2.0 \\ \alpha_n = 0.6 \pm 5.0 \end{cases}$
  - Quasi-free Compton scattering on deuterium:
    $\begin{cases} \alpha_n = 7.6 \pm 14.0 \\ \beta_n = 1.2 \pm 7.6 \\ \alpha_n - \beta_n = 9.8 \pm 3.6 \pm 2.2^{+2.1}_{-1.1} \end{cases}$

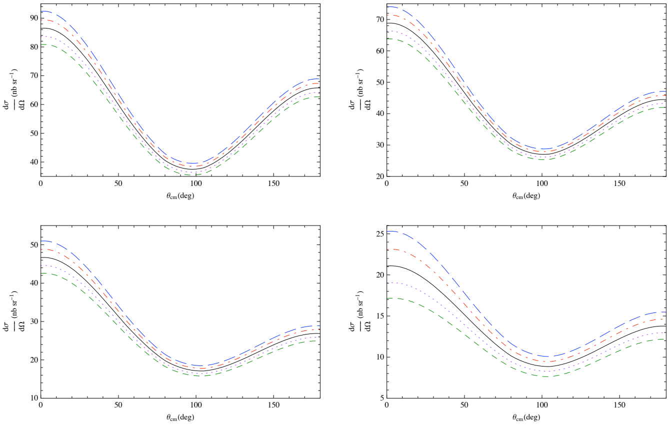
Compton scattering on ^{3}He

- HBChPT can be used in EFT calculations in few-body systems (χET)
- One-body operators from HBChPT
- Wave functions and two-body operators derived in EFT formalism
- Use ^{3}He to extract neutron polarizabilities
- Advantage: differential cross section large compared to deuteron case
- First theoretical treatment of Compton scattering on ^{3}He performed in χET
- Current results: $\mathcal{O}(q^3)$, no Δ
- Inclusion of Δ, extension to $\mathcal{O}(q^4)$ are ongoing

D. Shukla, A. Nogga and D. R. Phillips, Nucl. Phys. A **819**, 98 (2009)

Sensitivity to α_n

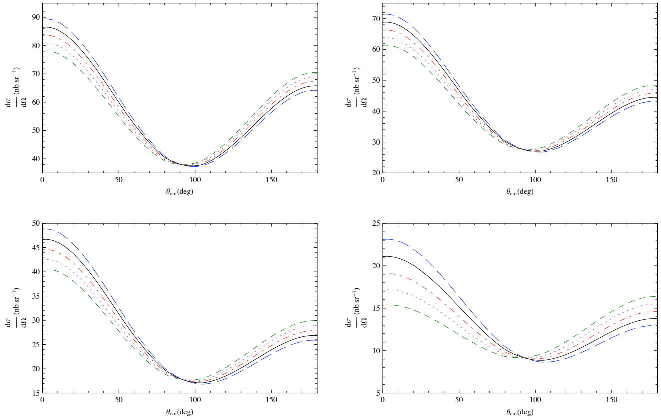
Differential cross section with varying α_n at different energies



Vary $\Delta\alpha_n = (-4, \ldots, 4) \times 10^{-4}\text{fm}^3$

Sensitivity to β_n

Differential cross section with varying β_n at different energies



Vary $\Delta\beta_n = (-2, \ldots, 6) \times 10^{-4} \text{fm}^3$

Spin polarizabilities

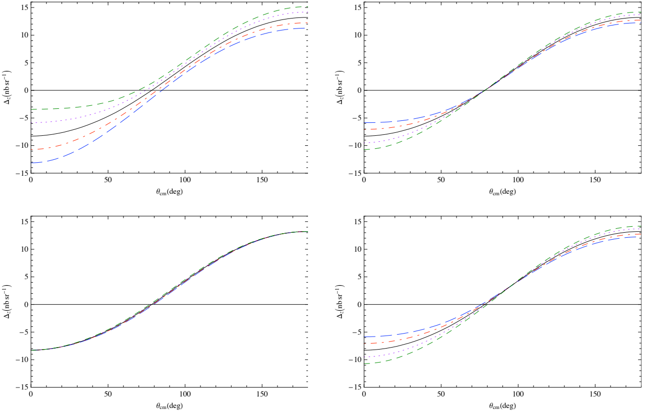
- In the following:

$$\gamma_{E1E1} = -\gamma_1 - \gamma_3, \quad \gamma_{M1M1} = \gamma_4$$
$$\gamma_{M1E2} = \gamma_2 + \gamma_4, \quad \gamma_{E1M2} = \gamma_3$$

- Very little information on spin polarizabilities γ_i
- Backward spin polarizability $\gamma_\pi = \gamma_1 + \gamma_2 + 2\gamma_4$: in agreement with HBChPT at order $\mathcal{O}(q^3)$ for proton and neutron
- Forward spin polarizability $\gamma_0 = \gamma_1 - (\gamma_2 + 2\gamma_4)$
- Can be accessed in double-polarization experiments
- Observables:

$$\Delta_z = \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\uparrow} - \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\downarrow}$$
$$\Delta_x = \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\rightarrow} - \left(\frac{d\sigma}{d\Omega}\right)_{\uparrow\leftarrow}$$

Δ_z in ^{3}He (120 MeV)



from top left to bottom right: vary $\gamma_{i,n}$, $i = 1, 2, 3, 4$ by $\pm 100\%$ of $\mathcal{O}(q^3)$ prediction